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ON POWER FUNCTION OF A CONDITIONALLY SPECIFIED TEST PROCEDURE IN AN UNBALANCED RANDOM EFFECTS MODEL*

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SUMMARY

This paper deals with a hypothesis testing problem in a two-stage unbalanced nested design with random effects. As an exact test of hypothesis of no treatment effect is impossible, Tan and Cheng [13] proposed an approximate *F*-test using Satterthwaite procedure. A conditionally specified test (CST) procedure is developed here using a preliminary test and the Tan and Cheng approximate *F*-test. The power function of the CST procedure is derived. The power gain of the CST procedure over the Tan and Cheng test procedure is studied.

Keywords : Conditionally specified test; preliminary test; unbalanced nested design; random effects.

Introduction

For testing hypothesis about certain main effect(s) in ANOVA, sometimes, no mean square is adequate as error mean square unless certain effects are zero. Therefore, we first test the significance of the doubtful effect(s) prior to the testing of main hypothesis. The test for the doubtful effect is called preliminary test. The procedure of testing a main

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hypothesis incorporating preliminary test (s)is called conditionally specified test (CST) procedure and it belongs to the area "Inference based on Conditional Specification" (Bancroft and Han [3]).

Some of the recent related papers in this area are Agarwal and Gupta [1], Ali and Srivastava [2], Girma Wolde-Tsadik and Afifi [5], Gupta and Gupta [6], Rao and Saxena [9], Rao and Saxena [10] and Singh and Saxena [12].

2. The Model Under Investigation and Conditionally Specified Test Procedure

Consider an experiment in which i female mice (dams) had been successively mated over a period of time with j males (sires). These j sires were different for every ith dam. The litters resulting from each mating were kept seperate and the blood pH of female litters was determined. The female offsprings were unequal in number for each combination of ith dam and jth sire. Further each dam was mated with unequal number of sires. The i dams and j sires were random samples drawn from large populations.

The above experiment is described by a two-stage unbalanced nested design. Let Y_{ijk} denote the blood pH reading of the kth female litter born from ith dam and jth sire mating combination. The sample observations may be represented by the following linear model

$$\left.\begin{aligned} &Y_{ijk} = \mu + d_i + s_{j(i)} + e_{k(i,j)} \\ &i = 1, \ldots, t;\, j = 1, \ldots, n_i;\, k = 1, \ldots, n_{ij}; \end{aligned}\right\} \tag{2.1}$$

where μ is the true overall mean, d_i is the effect of the ith dam, $s_{j(i)}$ is the effect of the jth sire mated with ith dam and $e_{k(i,j)}$ is the effect of the kth female litter born from ith dam and jth sire mating combination. The random variables d_i, $s_{j(i)}$ and $e_{k(i,j)}$ are normally and independently distributed with zero means and variances σ_d^2, σ_s^2 and σ^2 respectively.

The ANOVA resulting from model (2.1) is given in Table 2.1.

Here

$$c_1 = (N - N^*)/\nu_2,\ c_2 = (N^* - \sum_i \sum_j n_{ij}^2/N)|\nu_3,$$

$$c_3 = [N - \sum_i (\sum_j n_{ij})^2/N]/\nu_3,\ N = \sum_i \sum_j n_{ij}$$

TABLE 2.1—ANOVA FOR A TWO-STAGE UNBALANCED NESTED DESIGN WITH RANDOM EFFECTS MODEL

Source of variation	*Degrees of Freedom*	*Mean Square*	*Expected Mean Square*
Between dams (Treatments)	$\nu_3 = t - 1$	V_3	$\sigma_3^2 = \sigma^2 + c_2\sigma_s^2 + c_3\sigma_d^2$
Between sires within dams	$\nu_2 = \sum_i (n_i - 1)$	V_2	$\sigma_2^2 = \sigma^2 + c_1\sigma_s^2$
Between female litters within sires and dams	$\nu_1 = \sum_i \sum_j (n_{ij} - 1)$	V_1	$\sigma_1^2 = \sigma^2$

and $N^* = \sum_i (\sum_j n_{ij}^2 / \sum n_{ij}$.

Assume that the mean squares V_i ($i = 1, 2, 3$) in Table 2.1 are independently distributed as $\chi_i^2 \; \sigma_i^2/\nu_i$, where χ_i^2 is the central chi-square statistic with ν_i degrees of freedom.

One is interested in testing the hypothesis $H_0 : \sigma_d^2 = 0$ against the alternative $H_1 : \sigma_a^2 > 0$. On examination of the expected mean squares in Table 2.1, it is seen that an exact test of the hypothesis $H_0 : \sigma_d^2 = 0$ is impossible, but some approximate F-tests are available in the literature. Recently Tan and Cheng [13] proposed an F-statistic,

$$F = [V_3 + (c_2/c_1) V_1]/[(c_3/c_1) V_2 + V_1] \tag{2.2}$$

for testing H_0. Using Satterthwaite [11] prpcedure this F-statistic follows F-distribution approximately with ν_4 and ν_5 degrees of freedom, where

$$\nu_4 = (V_3 + \delta V_1)^2/(V_3^2/\nu_3 + \delta^2 V_1^2/\nu_1),$$

$$\nu_5 = (\delta V_2 + V_1)^2/(\delta^2 V_2^2/\nu_2 + V_1^2/\nu_1)$$

and $\delta = c_3/c_1$.

However, if we are in doubt whether $\sigma_s^2 \geqslant 0$, then before testing H_0 first test the preliminary hypothesis

$H_{10} : \sigma_s^2 = 0$ against the alternative

$H_{11} : \sigma_s^2 > 0.$

If this preliminary test turns out to be nonsignificant, use $F = V_3/V_{12}$ for testing H_0, where $V_{12} = \nu_1 V_1 + \nu_2 V_2)/(\nu_1 + \nu_2)$. On the other hand, if the preliminary test is significant, use $F = (V_3 + \delta V_1)/(\delta V_2 + V_1)$ for testing H_0. Under such circumstances, a test procedure known as conditionally specified test procedure is recommended which is obtained as a result of first testing H_{10} before the main test of H_0.

Conditionally Speeified Test (CST) Procedure

Reject $H_0 : \sigma_d^2 = 0$ if any one of the following two mutually exclusive

events occurs :

$$E_1 : \{V_2/V_1 > F_1, (V_3 + \delta V_1)/(\delta V_2 + V_1) > F_2\};$$

$$E_2 : \{V_2/V_1 \leqslant F_1, V_3/V_{12} \geqslant F_3\};$$

where

$$F_1 = F(\nu_2, \nu_1; \alpha_1),$$

$$F_2 = F(\nu_4, \nu_5; \alpha_2),$$

$$F_3 = F(\nu_3, \nu_1 + \nu_2; \alpha_3);$$

and $F(\nu_i, \nu_j; \alpha_k)$ denotes upper 100 α_k% point of F-distribution with ν_i and ν_j degrees of freedom. The values of F_2 for fractional ν_i's ($i = 4, 5$) are interpolated using Laubscher [7] interpolation formulae.

It may be remarked here that δ is not equal to unity in this unbalanced nested design. When $\delta = 1$ (i.e., $c_1 = c_2$), one may better use the exact test procedure given by Bozivich *et al.* [4].

3. Power of CST Procedure

The power of the CST procedure is given by

$$P = P_1 + P_2$$

where P_1 and P_2 are probabilities of the events E_1 and E_2.

3.1 Integral Expressions for Power Components

The joint density of V_1, V_2 and V_3 can be written as

$$h(V_1, V_2, V_3) = K V_1^{\frac{\nu_1}{2}-1} V_2^{\frac{\nu_2}{2}-1} V_3^{\frac{\nu_3}{2}-1}$$

$$X \exp\left\{-\frac{1}{2}\left(\frac{\nu_1 V_1}{\sigma_1^2} + \frac{\nu_2 V_2}{\sigma_2^2} + \frac{\nu_3 V_3}{\sigma_3^2}\right)\right\},$$

where K is a normalizing constant and is independent of V_i's.

Introducing new variates

$$u_1 = \frac{\nu_2 V_2}{\theta_{21} \nu_1 V_1},\ u_2 = \frac{\nu_3 V_3}{\theta_{32} \nu_2 V_2} \text{ and } w = \frac{\nu_1 V_1}{\nu_2}$$

where $\theta_{ij} = \sigma_i^2/\sigma_j^2\ (i > j)$; and integrating w from 0 to ∞, the joint density of u_1 and u_2 comes out as follows :

$$f(u_1, u_2) = K^* \frac{u_1^{\frac{\nu_2+\nu_3}{2}-1} u_2^{\frac{\nu_3}{2}-1}}{(1 + u_1 + u_1 u_2)^{\frac{(\nu_1+\nu_2+\nu_3)}{2}}} \quad (3.1.1)$$

where

$$K^* = \{\Gamma(\nu_1/2 + \nu_3/2)\}/\{\Gamma(\nu_1/2)\,\Gamma(\nu_2/2)\,\Gamma(\nu_3/2)\}.$$

Now, the integral expressions of the power components P_1 and P_2 come out as follows

$$P_1 = \int_{u_1=a}^{\infty} \int_{u_2=r(su_1+q)/u_1}^{\infty} f(u_1, u_2)\, du_1\, du_2 \quad (3.1.2)$$

$$P_2 = \int_{u_1=0}^{a} \int_{u_2=c(1+\theta_{11} u_1)/u_1}^{\infty} f(u_1, u_2)\, du_1\, du_2 \qquad (3.1.3)$$

where

$$a = (\nu_2 F_1)/(\nu_1 \theta_{21}), \quad c = (\nu_3 F_3/[(\nu_1 + \nu_2)\, \theta_{32}\theta_{21}]$$

$$q = \nu_2 (F_2 - \delta), \quad r = \nu_3/(\nu_1\, \nu_2\, \theta_{21}\theta_{33})$$

and $s = \theta_{21}\, \nu_1\, \delta\, F_2$.

3.2 Series Formulae for Power Components

In deriving the series formulae for P_1 and P_2, it is assumed that ν_i's ($i = 1, 2, 3$) are even integers. After deriving the integrals given in (3.1.2) and (3.1.3), the final expressions for P_1 and P_2 are obtained as follows :

$$P_1 = K^* \, S_{ij} \, \frac{B(X_1;\, \nu_1/2 + i - j,\ \nu_3/2 + j)}{(1 + qr)^{\nu_1/2 + i - j}\ (1 + rs)^{\nu_3/2 + j}} \qquad (3.2.1)$$

$$P_2 = K^* \, S_{ij} \, \frac{B(X_2;\, \nu_3/2 + j,\ \nu_1/2 + i - j)}{(1 + c)^{\nu_1/2 + i - j}\ (1 + c\theta_{21})^{\nu_3/2 + j}} \qquad (3.2.2)$$

where

$$S_{ij} = S_i \sum_{j=0}^{i} \binom{i}{i}, \quad S_i = \sum_{i=0}^{\nu_3/2 - 1} \frac{(-1)^i \binom{\nu_3/2 - 1}{i}}{\left(\frac{\nu_1 + \nu_2}{2} + i\right)},$$

$$X_1 = \frac{(1 + qr)}{1 + qr + (1 + rs)\, a}, \quad X_2 = \frac{(1 + c\theta_{21})\, a}{1 + c + (1 + c\theta_{21})\, a}$$

and $B\,(X;\, m, n)$ is an incomplete beta function with parameters m and n.

4. Discussion on Size and Power of CST Procedure

In this section the results of size and power of the CST procedure are discussed. From the series formulae given in (3.2.1) and (3.2.2), it is clear that the power of the CST procedure is a function of nine parameters: three degrees of freedom (ν_1, ν_2 and ν_3), one preliminary level of significance (α_1), two final levels of significance (α_2 and α_3), two variance ratios (θ_{21} and θ_{32}) and δ. The power of the CST procedure becomes size of the procedure when

$$\theta_{32} = \delta + (1 - \delta)/\theta_{21}. \tag{4.1}$$

An analytical study of the power function for variations of one or more of these parameters is not possible and hence we have resorted to an empirical investigation of the power function.

The degrees of freedom (ν_1, ν_2 and ν_3) and δ are completely determined by the experiment, while the variance ratios (θ_{21} and θ_{32}) are, in general, unknown to the experimenter and hence none of these six parameters are at the disposal of the experimenter. For convenience, the final levels of significance α_2 and α_3 are considered equal (i.e., $\alpha_2 = \alpha_3 = \alpha_f$) and chosen as $\alpha_f = 0.05$ before the experiment is done. Hence the preliminary level α_1 is the only parameter left at the disposal of the experimenter. Four values of α_1 ($\alpha_1 = 0.05$, 0.10, 0.25 and 0.50) are considered and the choice is made for that value of α_1 whieh yields

(i) size of our test procedure lies in the vicinity of prior fixed tolerance limit, and
(ii) gain in power of the CST procedure over the Tan and Cheng test procedure of the same size.

The numerical calculations for size and power are made with the help of a digital computer HCL WORKHORSE II.

4.1 Discussion on Size

The following five hypothetical sets of degrees of freedom for an exhaustive study of size and power have been considered :

	Degrees of freedom		
	ν_1	ν_2	ν_3
Set 1 :	6	4	2
Set 2 :	8	4	2
Set 3 :	8	6	2
Set 4 :	8	6	4
Set 5 :	4	4	2

Ten size tables (two tables for each set) are calculated for the above five sets of degrees of freedom, but only two tables are given for illustration (see Tables 1 and 2 in the Appendix). From all the size tables, it is observed that the size is not stationary but fluctuates considerably. Sometimes it is much larger than 0.05, the final level of significance. Of special interest is the size maximum for each of these size tables. It is found that the size maximum decreases as preliminary level increases when $\delta \geqslant 0.5$, while size maximum increases as preliminary level increases when $\delta < 0.5$. The location and magnitude of size maximum from all the size tables are compiled in Tables 3, 4 and 5 of the Appendix for $\alpha_1 = 0.25$, $\alpha_1 = 0.50$ and $\alpha_1 = 0.05$ respectively. From these tables we observe that the location of size maximum is not fixed. We observe from Tables 3 and 4 that the size maxima occur at $\delta = 1.5$, $\theta_{21} = 2.5$, $\theta_{32} = 1.3$ when $\delta > 0.5$ and $\alpha_1 = 0.25$ or $\alpha_1 = 0.50$. When $\delta \geqslant 0.5$, it is also observed that the size maximum lies in the neighbourhood of 0.11 with the application of $\alpha_1 = 0.25$ and the size maximum comes below 0.07 with the application of $\alpha_1 = 0.50$. We observe from Table 5 that the size maximum occurs at $\delta = 0.1$, $\theta_{21} = \theta_{32} = 1.0$ when $\delta < 0.5$ and $\alpha_1 = 0.05$. When $\delta < 0.5$, it also observed that the size maximum comes below 0.06 with the application of $\alpha_1 = 0.05$.

Now examine the effect of variation of degrees of freedom on size maximum when other parameters are kept constant. From Tables 3, 4 and 5, we observe that

(a) size maximum increases as ν_1 increases;
(b) size maximum decreases as ν_2 increases;
(c) size maximum increases as ν_3 increases.

4.2 *Discussion on Power and Power Gain*

For selected values of δ, θ_{21}, θ_{32} and $\alpha_1 = 0.05$, 0.25 and 0.50 we have calculated

(i) the size of CST procedure (here $\theta_{32} = \delta + (1 - \delta)/\theta_{21}$),
(ii) the power of CST procedure (here $\theta_{32} > \delta + (1 - \delta)/\theta_{21}$) and
(iii) the power of Tan and Cheng test procedure of the same size as obtained in (i) and for the same values of θ_{32} considered in (ii).

For a precise study, the power gain (i.e, power of CST procedure—power of Tan and Cheng test procedure) of the CST procedure over the

Tan and Cheng test procedure of the same size is obtained. The power gain is calculated for all the five sets of degrees of freedom but only three tables are given for illustration (see Tables 6, 7 and 8 in the Appendix). From all the power gain tables we observe the following:

Case (i) : $\delta \geqslant 0.5$ *and* $\alpha_1 = 0.25$ *and* 0.50

(1) The largest power gain appears at $\theta_{21} = 1.0$ and power gain, in general, decreases as θ_{21} increases for all values of δ and θ_{32}.
(2) Power gain, in general, increases first and then decreases as δ increases when $\theta_{21} = 1.0$ and for all values of θ_{32}.
(3) The CST procedure is more powerful than the Tan and Cheng test procedure in the following situations :
   (a) $\alpha_1 = 0.25, 0.5 \leqslant \delta \leqslant 1.5, \theta_{21} \leqslant 2.0$ and for all values of θ_{32}.
   (b) $\alpha_1 = 0.50, 0.5 \leqslant \delta \leqslant 1.5$ and for all values of θ_{21} and θ_{32}.
(4) There is a transition from power gain to power loss between $\theta_{21} = 2.0$ and $\theta_{21} = 3.0$ when $\alpha_1 = 0.25$.
(5) There is a general tendency that power gain first increases and then decreases as θ_{32} increases.

Case (ii) : $\delta < 0.5$ *and* $\alpha_1 = 0.05$

(1) The largest power gain appears at $\theta_{21} = 1.0$ and power gain decreases as θ_{21} increases for all values of δ and $\theta_{32} \geqslant 4.0$.
(2) Power gain increases as δ increases for all values of θ_{21} and θ_{32}.
(3) CST procedure is more powerful than the Tan and Cheng test procedure when $\theta_{21} \leqslant 3.0$ and for all values of θ_{32}. In general there is a transition from power gain to power loss between $\theta_{21} = 3.0$ and $\theta_{21} = 4.0$.
(4) There is a general tendency that power gain increases first and then decreases as θ_{32} increases.

5. Recommendations

In formulating recommendations based on the empirical results, it is tried to evolve the situations wherein the power of the CST procedure over the Tan and Cheng test procedure is more and the size remains in control. The experimenter is, therefore, advised to make use of the CST procedure or the Tan and Cheng test procedure by proceeding as follows :

1. If the experimenter can tolerate a size of 11% and if the following situations are satisfied

$$0.5 \leqslant \delta \leqslant 1.5,\ \theta_{21} \leqslant 2.0 \text{ and } \nu_3 \leqslant 4. \tag{5.1}$$

he is advised to use the CST procedure with preliminary level $\alpha_1 = 0.25$.

2. If the experimenter can not tolerate a size of 11%, but if he can tolerate a size of 7% and if the following situations are satisfied

$$0.5 \leqslant \delta \leqslant 1.5, \text{ for all } \theta_{21} \text{ and } \nu_3 < 4, \tag{5.2}$$

he is advised to use the CST procedure with preliminary level $\alpha_1 = 0.50$.

3. If the experimenter can tolerate a size of 6% and if the following situations are satisfied

$$\delta < 0.5,\ \theta_{21} \leqslant 3.0 \text{ and } \nu_3 \leqslant 4, \tag{5.3}$$

he is advised to use the CST procedure with preliminary level $\alpha_1 = 0.05$.

4. For all those situations other than the three situations given in (5.1), (5.2) and (5.3), the experimenter is advised to use the Tan and Cheng test procedure.

The above recommendations depend on an unknown parameter θ_{21}. However, in a practical situation one is confronted with the problem of using the approximation and merely replacing the unknown parameters by their observed estimates (see, e.g., Myers and Howe [8]). In a practical situation of this problem, V_2/V_1 is used as an estimate for θ_{21}.

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APPENDIX

TABLE 1—SIZE OF CST PROCEDURE FOR $\nu_1 = 6, \nu_2 = 4, \nu_1 = 2, \delta \geqslant 0.5$

δ	θ_{21}	θ_{32}	α_1 0.10	0.25	0.50
0.5	1.0	1.0000	.0491	.0473	.0441
	1.5	0.8333	.0540	.0513	.0471
	2.0	0.7500	.0582	.0542	.0489
	2.5	0.7000	.0614	.0560	.0497
	3.0	0.6667	.0637	.0570	.0499
0.7	1.0	1.0000	.0474	.0440	.0391
	1.5	0.9000	.0602	.0539	.0462
	2.0	0.8500	.0695	.0601	.0498
	2.5	0.8200	.0759	.0637	.0514
	3.0	0.8000	.0802	.0654	.0518
0.9	1.0	1.0000	.0465	.0419	.0354
	1.5	0.9667	.0675	.0580	.0464
	2.0	0.9500	.0823	.0676	.0518
	2.5	0.9400	.0920	.0728	.0541
	3.0	0.9333	.0979	.0751	.0547
1.1	1.0	1.0000	.0459	.0404	.0325
	1.5	1.0333	.0756	.0630	.0476
	2.0	1.0500	.0959	.0761	.0547
	2.5	1.0600	.1086	.0826	.0575
	3.0	1.0667	.1158	.0850	.0580
1.3	1.0	1.0000	.0456	.0394	.0304
	1.5	1.1100	.0841	.0687	.0495
	2.0	1.1500	.1099	.0851	.0582
	2.5	1.1800	.1252	.0925	.0612
	3.0	1.2000	.1333	.0949	.0616
1.5	1.0	1.0000	.0454	.0388	.0289
	1.5	1.1667	.0929	.0748	.0519
	2.0	1.2500	.1239	.0942	.0620
	2.5	1.3000	.1414	.1024	.0651
	3.0	1.3333	.1499	.1043	.0650

TABLE 2—SIZE OF THE CST PROCEDURE FOR $\nu_1 = 6, \nu_2 = 4, \nu_3 = 2, \delta < 0.5$

δ	θ_{21}	θ_{32}	α_1		
			0.05	*0.10*	*0.25*
0.1	1.0	1.0000	.0555	.0581	.0609
	1.5	0.7000	.0468	.0503	.0540
	2.0	0.5500	.0428	.0469	.0510
	2.5	0.4600	.0410	.0454	.0497
	3.0	0.4000	.0401	.0447	.0491
	4.0	0.3250	.0399	.0445	.0487
	5.0	0.2800	.0404	.0450	.0488
0.3	1.0	1.0000	.0513	.0519	.0523
	1.5	0.7667	.0487	.0498	.0507
	2.0	0.6500	.0483	.0496	.0506
	2.5	0.5800	.0487	.0500	.0509
	3.0	0.5333	.0493	.0506	.0512
	4.0	0.4750	.0507	.0516	.0514
	5.0	0.4400	.0517	.0522	.0511

TABLE 3—LOCATION AND MAGNITUDE OF SIZE MAXIMUM FOR $\delta \geqslant 0.5, \alpha_1 = 0.25$

Degrees of Freedom			*Location*			*Magnitude*
ν_1	ν_2	ν_3	δ	θ_{21}	θ_{32}	
6	4	2	1.5	3.0	1.3333	0.1043
8	4	2	1.5	3.0	1.3333	0.1064
8	6	2	1.5	2.5	1.3000	0.0919
8	6	4	1.5	2.5	1.3000	0.1115
4	4	2	1.5	3.0	1.3333	0.1005

TABLE 4—LOCATION AND MAGNITUDE OF SIZE MAXIMUM FOR

$\delta \geqslant 0.5,\ \alpha_1 = 0.50$

Degrees of Freedom			Location			Magnitude
ν_1	ν_2	ν_3	δ	θ_{21}	θ_{32}	
6	4	2	1.5	2.5	1.3000	0.0651
8	4	2	1.5	2.5	1.3000	0.0657
8	6	2	1.5	2.5	1.3000	0.0605
8	6	4	1.5	2.5	1.3000	0.0667
4	4	2	1.5	3.0	1.3333	0.0649

TABLE 5—LOCATION AND MAGNITUDE OF SIZE MAXIMUM FOR

$\delta < 0.5,\ \alpha_1 = 0.05$

Degrees of Freedom			Location			Magnitude
ν_1	ν_2	ν_3	δ	θ_{21}	θ_{32}	
6	4	2	0.1	1.0	1.00	0.0555
8	4	2	0.3	5.0	0.44	0.0567
8	6	2	0.1	1.0	1.00	0.0553
8	6	4	0.1	1.0	1.00	0.0565
4	4	2	0.1	1.0	1.00	0.0572

TABLE 6—POWER GAIN OF THE CST PROCEDURE OVER THE TAN AND CHENG TEST PROCEDURE OF THE SAME SIZE FOR

$\nu_1 = 8, \nu_2 = 6, \nu_3 = 4, \delta \geqslant 0.5, \alpha_1 = 0.25$

δ	θ_{21}	θ_{32}					
		2	4	6	8	10	12
0.5	1.0	.0606	.1346	.1311	.1279	.1054	.0889
	1.5	.0652	.1009	.0905	.0741	.0606	.0502
	2.0	.0573	.0736	.0643	.0488	.0389	.0306
	3.0	.0391	.0425	.0344	.0242	.0178	.0149
0.7	1.0	.0591	.1432	.1541	.1408	.1228	.1066
	1.5	.0625	.1079	.1017	.0866	.0706	.0605
	2.0	.0491	.0723	.0638	.0521	.0434	.0346
	3.0	.0255	.0316	.0445	.0219	.0170	.0147
0.9	1.0	.0520	.1406	.1570	.1471	.1318	.1155
	1.5	.0685	.1236	.1186	.1226	.0867	.0760
	2.0	.0408	.0634	.0586	.0520	.0456	.0394
	3.0	.0117	.0128	.0124	.0108	.0093	.0083
1.1	1.0	.0375	.1212	.1485	.1441	.1303	.1161
	1.5	.0463	.0950	.0951	.0845	.0736	.0634
	2.0	.0306	.0518	.0456	.0388	.0327	.0281
	3.0	.0023	−.0064	−.0058	−.0068	−.0061	−.0048
1.3	1.0	.0294	.1098	.1355	.1334	.1265	.1100
	1.5	.0422	.0895	.0920	.0843	.0719	.0627
	2.0	.0307	.0437	.0397	.0333	.0271	.0241
	3.0	−.0027	−.0220	−.0080	−.0254	−.0221	−.0190
1.5	1.0	.0237	.0867	.1148	.1171	.1089	.1007
	1.5	.0304	.0716	.0754	.0675	.0589	.0510
	2.0	.0150	.0192	.0126	.0089	.0052	.0037
	3.0	−.0060	−.0344	−.0442	−.0443	−.0380	−.0339

TABLE 7—POWER GAIN OF THE CST PROCEDURE OVER THE TAN AND CHENG TEST PROCEDURE OF THE SAME SIZE FOR

$\nu_1 = 8, \nu_2 = 6, \nu_3 = 4, \delta \geqslant 0.5, \alpha_1 = 0.50$

δ	θ_{21}	θ_{32}					
		2	4	6	8	10	12
0.5	1.0	.0530	.1214	.1008	.1198	.0991	.0841
	1.5	.0558	.0902	.0829	.0689	.0569	.0475
	2.0	.0499	.0688	.0619	.0476	.0383	.0341
	3.0	.0406	.0476	.0393	.0285	.0210	.0177
0.7	1.0	.0499	.1270	.1407	.1310	.1158	.1015
	1.5	.0517	.0966	.0949	.0829	.0687	.0596
	2.0	.0368	.0718	.0670	.0566	.0480	.0388
	3.0	.0325	.0492	.0660	.0370	.0296	.0251
0.9	1.0	.0422	.1219	.1414	.1358	.1240	.1102
	1.5	.0550	.1106	.1129	.1015	.0880	.0784
	2.0	.0347	.0665	.0678	.0630	.0564	.0495
	3.0	.0216	.0418	.0438	.0392	.0339	.0291
1.1	1.0	.0310	.1007	.1312	.1290	.1210	.1089
	1.5	.0339	.0819	.1152	.0849	.0767	.0680
	2.0	.0247	.0585	.0605	.0567	.0506	.0450
	3.0	.0128	.0131	.0377	.0347	.0308	.0275
1.3	1.0	.0198	.0993	.1136	.1168	.1131	.1015
	1.5	.0286	.0738	.0859	.0849	.0766	.0694
	2.0	.0227	.0499	.0582	.0569	.0516	.0475
	3.0	.0069	.0204	.0427	.0275	.0265	.0244
1.5	1.0	.0071	.0600	.0884	.0957	.0923	.0884
	1.5	.0178	.0541	.0674	.0674	.0635	.0577
	2.0	.0086	.0249	.0319	.0346	.0327	.0308
	3.0	.0036	.0091	.0150	.0170	.0197	.0186

TABLE 8—POWER GAIN OF THE CST PROCEDURE OVER THE TAN AND CHENG TEST PROCEDURE OF THE SAME SIZE FOR $\nu_1 = \nu,\ \nu_2 = 4,\ \nu_3 = 2,\ \delta < 0.5,\ \alpha_1 = 0.05$

δ	θ_{21}	θ_{32}					
		2	4	6	8	10	12
0.1	1.0	.0352	.0792	.0905	.0918	.0900	.0842
	1.5	.0457	.0730	.0780	.0730	.0660	.0594
	2.0	.0439	.0574	.0556	.0485	.0474	.0423
	3.0	.0249	.0292	.0204	.0187	.0180	.0155
	4.0	.0075	.0036	.0040	.0023	.0012	.0004
	5.0	−.0026	−.0045	−.0063	−.0065	−.0065	−.0064
0.3	1.0	.0412	.0965	.1026	.1154	.1124	.1072
	1.5	.0470	.0818	.0869	.0842	.0784	.0705
	2.0	.0443	.0648	.0654	.0595	.0556	.0500
	3.0	.0341	.0422	.0389	.0358	.0320	.0295
	4.0	.0278	.0331	.0283	.0248	.0219	.0187
	5.0	.0242	.0260	.0232	.0195	.0186	.0160